Exercise 21

Find the domain of $y = \dfrac{x+3}{4 - \sqrt{x^2 - 9}}$.

Solution

For a rational function, the denominator cannot be zero. Additionally, only the square root of a nonnegative number can be taken.

$$4 - \sqrt{x^2 - 9} \neq 0 \quad \text{and} \quad x^2 - 9 \geq 0$$

$$\sqrt{x^2 - 9} \neq 4 \quad \text{and} \quad x^2 \geq 9$$

$$x^2 - 9 \neq 16 \quad \text{and} \quad \sqrt{x^2} \geq \sqrt{9}$$

$$x^2 \neq 25 \quad \text{and} \quad |x| \geq 3$$

$$\sqrt{x^2} \neq \sqrt{25} \quad \text{and} \quad (x \geq 3 \quad \text{or} \quad x \leq -3)$$

$$|x| \neq 5 \quad \text{and} \quad (x \geq 3 \quad \text{or} \quad x \leq -3)$$

$$x \neq \pm 5 \quad \text{and} \quad (x \geq 3 \quad \text{or} \quad x \leq -3)$$

Therefore, the domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$.